the phase change data are shown in Fig. 3 for comparison with the phase change data. Thermocouple data are of course influenced by lateral conduction errors (15 to 20% for this case) and by errors due to peak heating not occurring exactly at a thermocouple location; however, the peak heating values from the thermocouple data as shown in Fig. 3 are not expected to be more than 30% low. The thermocouple data fall somewhat below the present *PC* data but are in substantially better agreement with the present *PC* data than that of Ref. 3.

These results show that quantitative peak heating data such as those caused by interfering flowfields can be measured using the phase change coating technique; this method also gives a more exact location of the peak. Caution should be used, however, when measuring high peaks resulting from high shear flows which might remove part or all of the coating and thus give a false impression of complete phase change. In addition, camera and lighting should be positioned so that the viewing angle is nearly normal to the melt surface.

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Correction Factor for Heat Flux in an Expansion Nozzle

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Introduction

WHEN calculating the local heat flux in a rocket-engine nozzle wall, a linear temperature distribution perpendicular to the wall surface is generally assumed.[1,2] This assumption is fully true only for a steady one-dimensional heat flow in a conductor with a temperature-independent thermal conductivity. This Note shows under what conditions the assumption of a linear temperature distribution will result in substantial errors. A correction factor is given to account for nonlinear temperature distribution.

Analysis

The expansion nozzle has the contour $y(x)$ on the hot-gas side, where x is the axis of rotation of the nozzle. The wall is assumed to be made of a material with a temperature-dependent thermal conductivity λ (T). Figure 1 shows a curved wall element at point x. The nozzle contour is described by inclination angle ψ, defined as

$$\psi = \text{arc tan } |dy/dx| \tag{1}$$

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The radius of curvature r of the nozzle contour is derived from

$$r = [1 + (dy/dx)^2]^{3/2} (d^2y/dx^2)^{-1} \tag{2}$$

where $r > 0$, if the concave side of the nozzle contour points to the positive direction of the curve normal. Using arc differential db, the heat-flow area in the element becomes

$$dA(z) = 2\pi (y + z\cos\psi) \frac{(r-z)}{r} db \tag{3}$$

If a linear temperature distribution $T(z)$ is assumed, the heat flux on the coolant side of the nozzle wall results from

$$q_{\text{lin}} = \lambda(T_{cw}) \frac{T_{gw} - T_{cw}}{s} \tag{4}$$

T_{gw} being the hot-gas side wall temperature, T_{cw} that on the coolant side, and s the wall thickness. By using correction factor C, the real heat flux on the coolant side is determined by

$$q = C\, q_{\text{lin}} \tag{5}$$

The heat-balance of the element may be written as

$$-dA(z)\lambda(T) \frac{dT}{dz} = q[dA(z)]_{z=s} \tag{6}$$

because temperature gradients perpendicular to z are negligible, especially in the throat area.[3] Combining Eqs. (3-6) results in

$$\int_{T_{gw}}^{T_{cw}} \lambda(T)dT = -C\lambda(T_{cw})(T_{gw}-T_{cw})(y+s\cos\psi)$$
$$\times \frac{(r-s)}{s} \int_0^s \frac{dz}{(r-z)(y+z\cos\psi)} \tag{7}$$

Finally, dimensionless variables are introduced

$$Y = y/y_t \tag{8a}$$
$$R = r/y_t \tag{8b}$$
$$S = s/y_t \tag{8c}$$

where y_t is the radius of the nozzle throat. The influence of temperature on λ is given by the dimensionless parameter

$$m = \lambda(T_{gw})/\lambda(T_{kw}) \tag{9}$$

If λ (T) in Eq. (7) is replaced by a linear function and Eqs. (8)

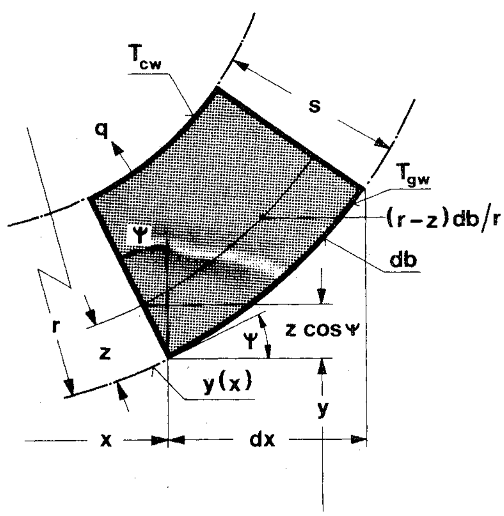

Fig. 1 Curved wall element for considering the heat balance.

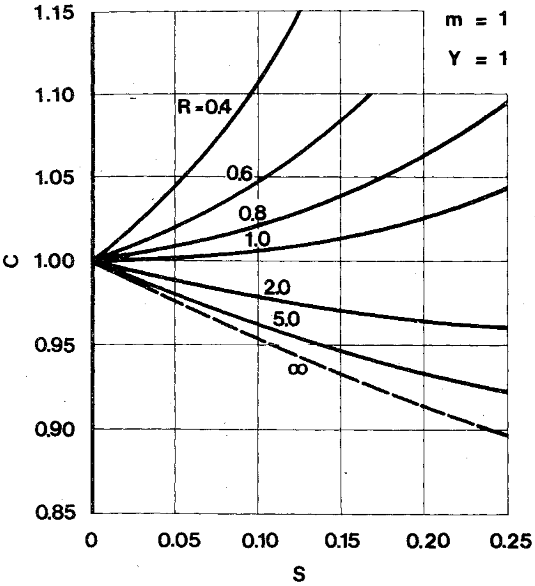

Fig. 2 Effect of nozzle geometry on correction factor.

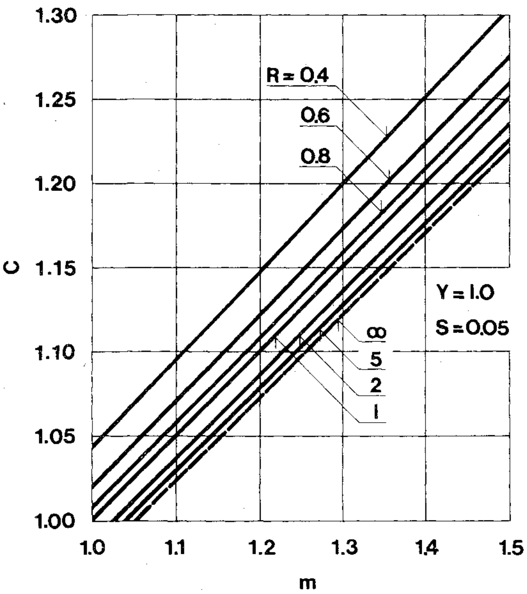

Fig. 3 Effect of temperature dependence of thermal conductivity on the correction factor.

and (9) are applied, the correction factor takes the form

$$C = \frac{S(m+1)\,(Y+R\cos\psi)}{2(R-S)\,(Y+S\cos\psi)} \left[\ln \frac{R(Y+S\cos\psi)}{Y(R-S)}\right]^{-1} \tag{10}$$

Geometrically similar expansion nozzles have identical correction factors if their temperature at corresponding points does not differ.

Discussion

As an example, the results for the particularly interesting nozzle throat ($Y=1$) are presented. In Fig. 2, C is plotted as a function of S with R as a parameter for $m=1$. It can be seen that for growing S the correction factor C deviates increasingly from the limiting case $C=1$. For expansion nozzles of rocket engines, $1 \le R \le 2$ and $S \le 0.05$ is generally recommended.[4] In this range there is apparently no need for correcting Eq. (4) because the error is $f<1\%$. Recently, however, nozzle geometries approaching $R \ge 0.5$ have been discussed.[5] The dashed line for $R \to \infty$ as a limiting case provides the correction factor for a tube.

Figure 3 shows correction factor C as a function of m for different values of R for $Y=1$ and $S=0.05$. This shows that frequently a correction of Eq. (4) becomes indispensable because the error reaches a percentage that cannot be tolerated even in approximate calculation.

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Errata

Economical Scheme for Estimating Orbital Lifetimes

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On page 324, Fig. 2 the curves labeled $e_0=0.7$ should be labeled $e_0=0.6$ and the curve labeled $e_0=0.60$ should be labeled $e_0=0.40$.

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